On the Refraction and Dispersion of Krypton and Xenon and their Relation to those of Helium and Argon.

By C. Cuthbertson, Fellow of London University, University College; and M. Cuthbertson.

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By the kindness of Sir William Ramsay and Prof. R. B. Moore, we have been enabled to measure the refractive indices of krypton and xenon with much larger quantities of these gases than were available at the time of their first isolation. The method of preparation of the gases will be given by Prof. Moore in a paper about to be presented to the Chemical Society. The procedure followed in determining the indices is described in a paper lately published by the Royal Society,* and need be only briefly recapitulated.

Jamin's refractometer was used, and the source of light was, for the refraction, a Bastian mercury lamp, and for the dispersion, a Nernst lamp in conjunction with a fixed-deviation spectroscope. In measuring the dispersion, one of the two tubes was filled with the gas in question, and the other with air at such a pressure that the optical lengths of path of the two interfering rays were approximately equal. The light was then continuously changed from $\lambda = 6500$ to $\lambda = 5000$, and the change in the position of the centre of a given bright band was observed in a telescope fitted with a micrometer eye-piece. The distance on the same scale between the centres of two adjacent bright bands of known wave-length (generally $\lambda = 5461$) was also noted. These observations constituted the point of departure. The pressure of the gas in one tube was then altered till a convenient number of bands of wave-length 6500 (usually 85) had passed in one direction; and, next, the pressure of air in the other tube was altered till the same number of bands had passed in the contrary direction. The wave-length of the light was then changed from the first wave-length to the second, and the movement of the centre of the bright band under observation was again noted.

Suppose that this movement is a divisions of the micrometer, and that the distance between the centres of two adjacent bright bands of wave-length 5461 is b divisions. Let N_R be the number of bands of wave-length 6500 which pass and repass, N^A_V the number of bands of wave-length 5000 which would have been observed with light of this wave-length if the pressure of the air alone had been changed, and N_V^G the corresponding number if the pressure of the gas alone had been changed.

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* "On the Dispersion of Gaseous Mercury, Sulphur, Phosphorus, and Helium," by C. Cuthbertson and E. P. Metcalfe, 'Roy. Soc. Proc.,' A, vol. 80, p. 411, 1908.

Then what is observed is that the relative path retardation

$$5000N_V^A - 5000N_V^G = \frac{a}{b}5461,$$

which may be written

$$\frac{5000N_V^A - 6500N_R}{\lambda N_\lambda} - \frac{5000N_V^G - 6500N_R}{\lambda N_\lambda} = \frac{a}{b} \cdot \frac{5461}{\lambda N_\lambda},$$

where λN is the path retardation for any wave-length. And since λN is in each case proportional to $\mu - 1$, this equation becomes

$$\frac{\mu_V^A - \mu_R^A}{\mu_\lambda^A - 1} - \frac{\mu_V^G - \mu_R^G}{\mu_\lambda^G - 1} = \frac{a}{b}\frac{5461}{\lambda N_\lambda}.$$

The first term on the left is the dispersive power of air, which is known, and since λN_λ is observed, the dispersive power of the gas can be calculated.

Krypton.—About 100 c.c. of gas at normal temperature and pressure were employed. The refractive index for the mercury green ray was determined six times, each determination being the mean of two readings of 100 bands, in one of which pressure was rising and in the other falling, so as to eliminate the effect of a drift of zero. The following are the values obtained :—

No. of experiment.	$(\mu - 1) 10^6$. $\lambda = 5461$.
1	428·71
2	428·64
3	428·86
4	428·15
5	428·92
6	428·57
Mean of 1, 2, 3, 5, and 6 ...	428·74

The divergence shown in the fourth experiment was accounted for, and it may be neglected. The others agree to one part in a thousand, and their mean may be accepted.

The dispersion was calculated from seven experiments :—

No. of experiment.	Excess of dispersive power of Kr over that of dry air.
1	0·00208
2	0·00204
3	0·00213
4	0·00207
5	0·00243
6	0·00191
7	0·00220
Mean.........	0·00212

The dispersive power of air has been investigated several times, but, unfortunately, the results are not very concordant. The following table shows the principal determinations, the difference of refractivity between $\lambda = 6500$ and $\lambda = 5000$, and the fraction expressing the dispersive power calculated for these limits.

Table I.

Observers.	Refractive index of air. $\mu - 1$.	$\mu_{5000} - \mu_{6500}$.	Dispersive power. $\dfrac{\mu_{5000} - \mu_{6500}}{\mu_{\infty} - 1}$.
Kayser and Runge...	$0{\cdot}00028817 + \dfrac{1{\cdot}316}{\lambda^2} + \dfrac{31600}{\lambda^4}$	0·00000246	0·0085481
Mascart	$0{\cdot}00028715 \left(1 + \dfrac{5{\cdot}8}{\lambda^2 10^{11}}\right)$	0·00000273	0·00947
Scheel	$0{\cdot}00028705 \left(1 + \dfrac{5{\cdot}67}{\lambda^2 10^{11}}\right)$	0·00000266	0·00926
Perreau	$0{\cdot}0002879 \left(1 + \dfrac{5{\cdot}6}{\lambda^2 10^{11}}\right)$	0·00000263	0·00914

The figures of Mascart, Scheel, and Perreau, are so nearly alike that the truth probably lies near them, and we select those of Scheel. According to him, the dispersive power of air between the limits $\lambda = 6500$ and $\lambda = 5000$ is 0·00926; and this, added to the excess of the dispersive power of krypton over air (0·00212) gives 0·01138.

If the dispersion of krypton is expressed in the form

$$\mu - 1 = a(1 + b/\lambda^2),$$

we have, therefore, $$b\left(\frac{1}{5000^2} - \frac{1}{6500^2}\right) = 0{\cdot}001138.$$

And from the refractivity for $\lambda = 5461$ we have

$$a\left(1 + \frac{b}{5461^2}\right) = 0{\cdot}0004287.$$

Hence $$\mu - 1 = 0{\cdot}0004189\left(1 + \frac{6{\cdot}97}{\lambda^2 10^{11}}\right).$$

It must be remembered that these figures are for the gas at normal temperature and pressure, without making allowance for the fact that the molecule is monatomic. In making comparison for an equal number of atoms (Kr_2), instead of an equal pressure, the value of a must be doubled.

In the first four experiments, readings were taken of the movement of the bands at the point $\lambda = 5800$. The following table shows the results :—

Table II.

	Dispersive power.		
	$\lambda = 6500$ to $\lambda = 5800$.	$\lambda = 5800$ to $\lambda = 5000$.	$\lambda = 6500$ to $\lambda = 5000$.
Excess of krypton over air ……	0·000707	0·001365	0·002072
Air (Scheel)………………………	0·003420	0·005840	0·00926
Krypton ………………………	0·004127	0·007205	0·001133
b (calculated) ……………………	6·9	7·0	6·937

It will be observed that the values of b calculated from the two sets of figures agree well.

Xenon.—About 120 c.c. of this gas were employed. The following table shows the results of four determinations of the refractive index for the green mercury line:—

No. of experiment.	$(\mu - 1) 10^6$.
1 ……………………	705·72
2 ……………………	705·30
3 ……………………	705·19
4 ……………………	705·74
Mean	705·49

The dispersion was calculated from eight experiments.

No. of experiment.	Excess of dispersive power over dry air.
1 ……………………	0·00645
2 ……………………	0·00732
3 ……………………	0·00800
4 ……………………	0·00728
5 ……………………	0·00750
6 ……………………	0·00749
7 ……………………	0·00680
8 ……………………	0·00758
Mean ………	0·00730
Add dispersive power of air =	0·00926
	0·01656

From these figures we find that the refractive index of xenon doubled (X_2) is expressed by the formula

$$\mu - 1 = 0{\cdot}0013646 \left(1 + \frac{10{\cdot}14}{\lambda^2 10^{11}}\right).$$

With regard to the accuracy of these results, it is only necessary to observe briefly that it depends on four principal factors:—

(1) The purity of the gas, which is dealt with in Prof. Moore's paper, to which reference has been made.

(2) The correctness of the assumed value for the dispersion of air: as shown above, this is probably trustworthy within 2 or 3 per cent.

(3) The experimental errors, which can be judged from the figures given. We think that they do not amount to much more than in the case of air.

(4) The adequacy of Cauchy's formula: the figures given for the intermediate values of the dispersion of krypton show that the degree of accuracy attained was not sufficient to test this point.

Relative Refractivities of the Inert Gases.

Helium.—The index of helium has recently been determined by four sets of observers. Their results are as follows, and are for He_1:—

Table III.—Refraction and Dispersion of Helium.

λ.	$\mu-1$.	Observers.
	$0\cdot00003478\left(1+\frac{2\cdot2}{\lambda^2 10^{11}}\right)$	W. Burton, 'Roy. Soc. Proc.,' A, vol. 80, 1908, p. 390.
0·0579 0·0576	0·000034384	Kurt. Herrmann, 'Ber. d. Deu. Phys. gesell.,' 1908, part 5.
0·05461	0·000034525	
0·04359	0·000035332	
0·06676	0·0000346	K. Scheel and Schmidt, *ib.*
0·05876	0·0000340	
0·05790	0·0000338	
0·05770	0·0000338	
0·05016	0·0000336	
0·04358	0·0000342	
∞	0·0000340 ± 2	
	$0\cdot0000347\left(1+\frac{2\cdot4}{\lambda^2 10^{11}}\right)$	Cuthbertson and Metcalfe, 'Roy. Soc. Proc.,' A, vol. 80, 1908, p. 411.

The first and last sets of figures agree very well; the others do not. We select those of Mr. Burton for comparison with our results.

Neon.—We regret that we have not been able to obtain a sufficient quantity of pure neon to enable us to measure its dispersion. The only value with which we are acquainted is that of Ramsay and Travers for white light; $\mu = 1\cdot0000687$, while, for air, $\mu = 1\cdot000293$.

Argon.—Mr. Burton has also redetermined the index of argon, finding for A_1 the value

$$\mu-1 = 0\cdot0002792\left(1+\frac{5\cdot6}{\lambda^2 10^{11}}\right).$$

On several occasions it has been shown how closely the refractivities of these gases approach to the ratios of 1, 2, 8, 12, and 20. We are now able to compare the values for infinite wave-lengths as calculated from Cauchy's formula, and find that the coincidence is still more striking. The original figures of Ramsay and Travers are shown in column 2. They are for white light. In columns 5, 6, and 7 the figures for neon are the old values for white light multiplied by two.

Table IV.—Relative Refractivities of the Inert Gases for infinite Wave-lengths.

1.	2.	3.	4.	5.	6.	7.
	Ramsay and Travers, 1901 (molecular). $(\mu-1)\,10^6$.	Empiric ratios.	Divergence from Col. 3, per cent.	Revised values (atomic). $(\mu-1)\,10^6$.	Col. 5 divided by Col. 3.	Divergence per cent. from Col. 3, revised values.
Helium	36·3	1	+2·3	69·56	69·56	−0·34
Neon	68·7	2	−3·2	(137·4)	68·7	−1·6
Argon	284	8	—	558·4	69·8	— standard
Krypton	425	12	−0·2	837·8	69·8	0·0
Xenon	689	20	−3·0	1364·6	68·23	−2·25

Relative Dispersive Power of the Inert Gases.

In his paper, cited above, Mr. Burton compares the dispersive powers of helium and argon with hydrogen, and points out that when they are expressed in the form $\mu-1 = a(1+b/\lambda^2)$ the values of a/b are roughly in the ratios of 1, 3, and 2 respectively. It is now possible to extend this comparison to krypton and xenon.

The following figures show that the coincidence fails in these cases. If the value of a/b for helium be taken as 1 and that for argon as 3, the values for krypton and xenon are not in the ratios of simple integers. It should also be observed that Mascart's value of b for hydrogen, adopted by Burton for comparison, is not supported by the determinations of other observers.*

	$10^6 a/b$.
Helium	31·62
Argon	99·72
Krypton	120·2
Xenon	134·5

Another relation between the refraction and dispersion can, however, be deduced. The accompanying diagram shows the values of b plotted against the refractivities for infinite wave-lengths given in Table IV.

A very good straight line can be drawn through the points representing helium, krypton, and xenon, and argon is not far off. If, instead of Burton's value for b for helium, we take Cuthbertson and Metcalfe's (2·4) the position of argon is improved. Burton states that his argon was prepared by Sir William Ramsay and obtained from Messrs. Tyrer and Co. He does not give its density. If it contained any nitrogen this would account for the large value of b, while the value of a would scarcely be affected. Considering

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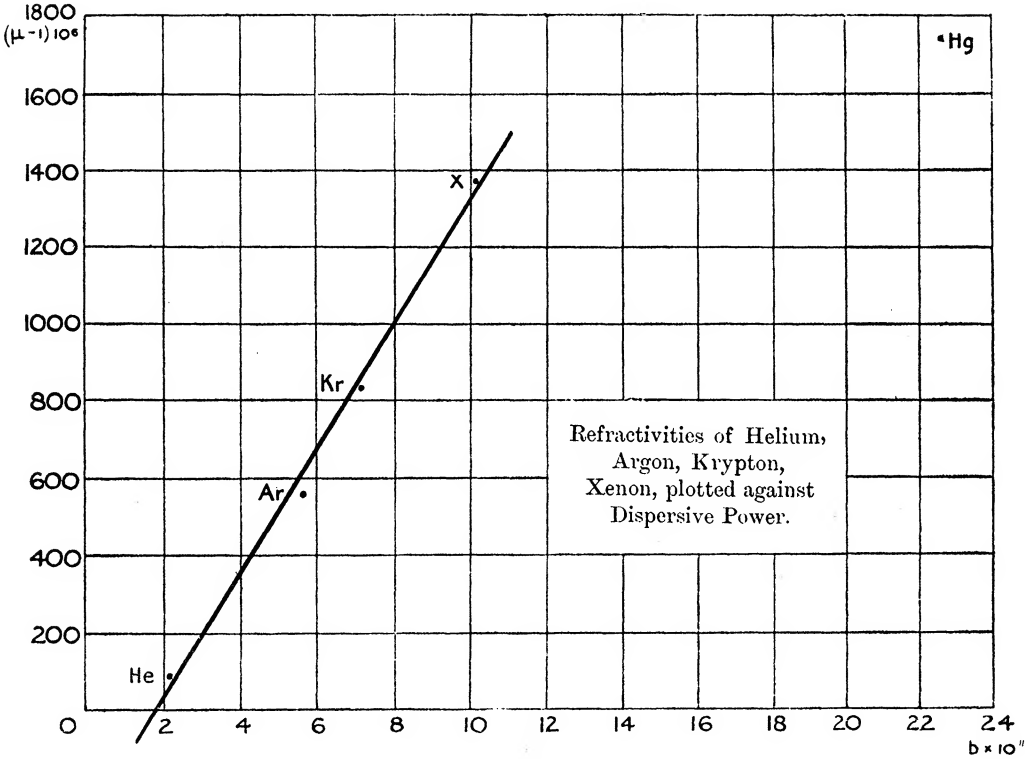
* Mascart's value is 4·3; from Ketteler's measurements we have 6·69, from Lorenz 7·55, from Perreau 7·54, from Scheel 7·52.



The dots for X, Kr, and He should be closer to the line.

that, even for air, the values of *b* have varied so widely as from 5·8 (Mascart) to 5·2 (Kayser and Runge), it may fairly be said that within the limits of experimental error *b*, in the inert gases, is a linear function of the refractivity for infinite wave-lengths.

It is unfortunate that, at present, we are unable to compare with these results the dispersions of any other series of more than two elements. The dispersions of selenium and tellurium and of arsenic have been roughly measured, but not with sufficient accuracy to justify their use.* But in the lighter members of the series to which they respectively belong the dispersions have been observed with somewhat greater accuracy, though the experimental results still leave much to be desired.†

The numbers found for nitrogen (Scheel) are

$$\mu-1 = 0\cdot00029061\left(1+\frac{7\cdot70}{\lambda^2 10^{11}}\right),$$

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* Cuthbertson and Metcalfe, 'Phil. Trans.,' A, vol. 207, p. 135.

† Cuthbertson and Metcalfe, "Dispersion of Gaseous Mercury, Sulphur, Phosphorus, and Helium," 'Roy. Soc. Proc.,' A, vol. 80, 1908.

and for phosphorus (Cuthbertson and Metcalfe)

$$\mu-1 = 0\cdot001162\left(1+\frac{15\cdot3}{\lambda^2 10^{11}}\right).$$

Similarly, the figures for oxygen (Mascart) are

$$\mu-1 = 0\cdot0002663\left(1+\frac{5\cdot07}{\lambda^2 10^{11}}\right),$$

and for sulphur (Cuthbertson and Metcalfe)

$$\mu-1 = 0\cdot0010457\left(1+\frac{21\cdot25}{\lambda^2 10^{11}}\right)$$

If we assume that in each group $b = p+qa$, where p and q are constants for the group, the values of p and q calculated from these figures are :—

	p.	q.
Inert gases (calculated from Kr and X)	$\frac{2\cdot07}{10^{11}}$	$\frac{0\cdot615}{10^7}$
Oxygen and sulphur	$\frac{-0\cdot45}{10^{11}}$	$\frac{2\cdot075}{10^7}$
Nitrogen and phosphorus	$\frac{5\cdot15}{10^{11}}$	$\frac{0\cdot875}{10^7}$

These numbers do not show any obvious relation, and more work is required before the law can be traced further. Thus, for example, the value of b for air ($5\cdot67/10^{11}$) cannot be deduced from the values for nitrogen; ($b = 7\cdot7/10^{11}$) and oxygen ($b = 5\cdot07/10^{11}$) here adopted from observers of high reputation. Mascart found for nitrogen the values 5·2, 7·6, and 7·3 in different parts of the spectrum, and pointed out that his values of b for nitrogen and oxygen could not be reconciled with his value for air, and that more accurate determinations were required.

It should, however, be remarked that, though the values of b for helium, krypton, and xenon obtained by us were calculated with reference to air, the linear relation which we find would not be affected by any change in the number adopted for the dispersion of air, which would increase or diminish all alike. A large error would affect their relation to Mr. Burton's values; but a large error is not to be feared.

In the diagram, the position of mercury is indicated, and it is plain that though it resembles the inert gases in being monatomic in the gaseous state, it does not fall into line with them in regard to the relation between refractive and dispersive power.

It remains to consider the physical meaning of the law deduced from our experiments. On comparing Cauchy's formula $\mu-1 = a(1+b/\lambda^2)$ with the theoretical expression

$$\mu-1 = \Sigma\, C/(n_0^2-n^2),$$

and making the assumption that the latter has only one term, for which n_0^2 is large compared with n^2, we have

$$a = \mathrm{C}/n_0^2, \qquad b = \lambda_0^2.$$

If $b = p + qa$ for all the inert gases it would follow that the square of the wave-length of the free vibration dominating the dispersion is equal to a constant plus a term proportional to the refractivity for infinite wave-length.

We have great pleasure in expressing our thanks to Sir William Ramsay and Prof. R. B. Moore for the loan of the gases; to Prof. Trouton, Prof. Porter, and the staff of the Physical Laboratory of University College, London, for much assistance and advice; and to the Royal Society for a grant in aid of these experiments.

Note on Tidal Bores.

By Lord Rayleigh, O.M., Pres. R.S.

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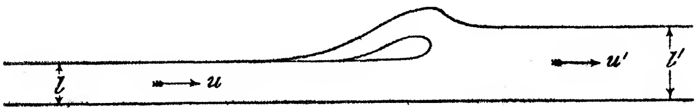
It was shown long ago by Airy that when waves advance over shallow water of depth originally uniform, the crests tend to gain upon the hollows,[*] so that the anterior slopes become steeper and steeper. Ultimately, if the conditions are favourable, there formed what is be may called a *bore*. Ordinary breakers upon a shelving beach are of this character, but the name is usually reserved for tidal bores advancing up rivers or estuaries. Interesting descriptions of some of these are given in Sir G. Darwin's 'Tides' (Murray, 1898).

Although the real bore advances up the channel, we may for theoretical purposes "reduce it to rest" by superposing an equal and opposite motion upon the whole water system. We have then merely to investigate the transition from a relatively rapid and shallow stream of depth l and velocity u to a deeper and slower stream of depth l' and velocity u' (fig. 1). The

Fig. 1.



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[*] See also 'Scientific Papers,' vol. 1, p. 253, 1899.